A CONTRAST OF TWO SYSTEMS OF CONTRAST CODING

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Introduction

Contingency tables have been extensively used to interpret the effects of independent factors on the dependent variables in categorical data analysis. Pioneering studies on contingency table analysis have been conducted by Neyman,[1] Goodman and Kruskal[2] and Kendall.[3] In addition, dummy variable coding systems have been used in contingency table analysis by Snedecor,[4] Cochran and Cox[5] and Grizzle, Stamer and Koch(abbreviated as GSK).[6]

The basic technique for using the dummy variable coding system is to establish an analysis of variance design matrix of dummy effects corresponding to the independent factors. The most commonly used dummy variables are coded either "0" and "1" or "-1" and "1". These two coding systems, however, can give a different interpretation between variables. The objectives of this study are to: (1) explicate mathematically the relationship between the two coding systems with and without interactions, and (2) demonstrate a proper interpretation of the variance analysis of these coding systems using data from a study of school desegregation.

Analysis of Techniques

For simplicity, only one observation in each cell of a contingency table is used in this study. It should be noted that the results are equivalent to those obtained using more than one observation in each cell. For convenience, the (-1,1) and (0, 1) coding systems are called code A and code B, respectively.

Two Way Classification Without Interaction (TWOI)

Suppose one wishes to study the relationship between political protest, income and education. Let y_{ij} be the proportion of protestors to nonprotestors for school desegregation in (i,j)th cell. The regression model can be written as

$$y_{ij} = \mu X_0 + \beta_1 X_{1i} + \beta_2 X_{2j} + \varepsilon_{ij} \quad \text{for code B} \qquad (1)$$

$$y_{ij} = \mu X_0 + \alpha_1 X_{1i} + \alpha_2 X_{2j} + \varepsilon_{ij} \quad \text{for code A} \qquad (2)$$

where, $X_0 = 1$; μ, β_k and α_k are regression coefficients, $k=1,2$; and ε_{ij} represents random error for the (i,j)th cell; $i, j = 1, 2$. The design matrix X and cell models are shown in Table 1.

Regression coefficients can be estimated as

$$b_1 = \tfrac{1}{2}(y_{2\cdot}-y_{1\cdot}),\ b_2 = \tfrac{1}{2}(y_{\cdot 2}-y_{\cdot 1})$$
$$a_1 = \tfrac{1}{4}(y_{2\cdot}-y_{1\cdot}),\ a_2 = \tfrac{1}{4}(y_{\cdot 2}-y_{\cdot 1}) \qquad (3)$$

where b_i is an estimator for β_i and a_i is an estimator for α_i, $i = 1, 2$. As Equation 3 shows,

$$b_i = \tfrac{1}{2}a_i, \quad i = 1, 2.$$

To test H_0: $\beta_i = 0$ or $\alpha_i = 0$, the Chi-square statistic with 1 df is

$$\chi^2_{\beta_i} = b_i^2/c_i \ , \quad \chi^2_{\alpha_i} = a_i^2/d_i \qquad (4)$$

where c_i and d_i denote the diagonal elements of $(X^TX\text{-}1)$ with respect to b_i and a_i respectively, and in which X is a design matrix in the model. Substituting $c_i = 1$ and $d_i = \tfrac{1}{4}$, computed from Table 1(i), into Equation 4 yields

$$\chi^2_{\beta_i} = b_i^2/1 = (2a_i)^2/1 = a_i^2/\tfrac{1}{4} = \chi^2_{\alpha_i} \qquad (5)$$

This result implies that either code B or code A gives the same statistical analysis except that the magnitude of the regression coefficients in code B is twice as large as in code A.

Two Way Classification With Interaction (TWI)

If one wishes to study income and education and their interaction effects on protest, then the cell observations, y_{ij}, can be defined as

$$y_{ij} = \mu X_0 + \beta_1 X_{1i} + \beta_2 X_{2j} + \beta_{12} X_{1i} X_{2j} + \varepsilon_{ij} \qquad (6)$$
for code B

$$y_{ij} = \mu X_0 + \alpha_1 X_{1i} + \alpha_2 X_{2j} + \alpha_{12} X_{1i} X_{2j} + \varepsilon_{ij} \qquad (7)$$
for code A

where μ, α_i and β_i are defined in the section TWOI; β_{12} and α_{12} are interaction regression coefficients. The design matrix X and cell models are shown in Table 1(ii).

From Table 1(ii) the regression coefficients can be derived as follows

$$b_1 = y_{21} - y_{11}\ , \quad b_2 = y_{12} - y_{11}$$
$$b_{12} = (y_{11} + y_{22}) - (y_{12} + y_{21})$$
$$a_1 = \tfrac{1}{4}(y_{2\cdot} - y_{1\cdot}), \quad a_2 = \tfrac{1}{4}(y_{\cdot 2} - y_{\cdot 1}) \qquad (8)$$
$$a_{12} = \tfrac{1}{4}[(y_{11} + y_{22}) - (y_{12} + y_{21})]$$

in which a_i, b_i, a_{12} and b_{12} are estimators of α_i, β_i, α_{12} and β_{12}, respectively; and where $i = 1, 2$.

As Equation 8 shows, the relationships between a_i and b_i are:

$$a_1 = \tfrac{1}{2}b_1 + \tfrac{1}{4}b_{12}\ , \quad a_2 = \tfrac{1}{2}b_2 + \tfrac{1}{4}b_{12}$$
$$a_{12} = \tfrac{1}{4}b_{12} \qquad (9)$$

Equation 8 indicates that the b_i are only testing the main effect in lower level groups. That is, b_1 is testing the income effect in the low education group, and b_2 is testing the education effect in the low income group. However, the a_i are testing main effects over all levels. Both b_{12} and a_{12} are testing the interaction effects between income and education over all levels.

As Equation 9 shows, the Chi-square statistics for testing main effects in $\beta_1 = 0$ and $\beta_2 = 0$ are different from those of testing in $\alpha_1 = 0$ and $\alpha_2 = 0$. However, the testing for $\beta_{12} = 0$ and $\alpha_{12} = 0$ are equivalent. The diagonal elements of $(X^TX)^{-1}$ with respect to β_{12} and α_{12} equal 4 and $\tfrac{1}{4}$, respectively, such that

$$\chi^2_{\beta_{12}} = b_{12}^2/4 = (4a_{12})^2/4 = a_{12}^2/\tfrac{1}{4} = \chi^2_{\alpha_{12}} \qquad (10)$$

These results imply that if interaction effects are present in the model, the analysis should be interpreted carefully because results obtained by the two coding systems are not identical with respect to the main effects. For example, if b_1 is significant, it should be concluded that the income effect is significant in the lower education group. However, the a_1 can be used to test

TABLE 1 (i), (ii)

TWO WAY CLASSIFICATION DESIGN MATRIX AND CELL MODEL

Coding		(0, 1) B				(-1, 1) A			
Variable \ Model		(ii) TWI: (i) TWOI		(ii) TWI: (i) TWOI		(ii) TWI: (i) TWOI		(ii) TWI: (i) TWOI	
Income	Education	X_0 X_1 X_2	X_{12}	Cell Model		X_0 X_1 X_2	X_{12}	Cell Model	
Low	Low	1 0 0	0	$y_{11} = \mu + \varepsilon_{11}$		1 -1 -1	1	$y_{11}=\mu-\alpha_1-\alpha_2+\varepsilon_{11}$	$+\alpha_{12}$
Low	High	1 0 1	0	$y_{12} = \mu + \beta_2 + \varepsilon_{12}$		1 -1 1	-1	$y_{12}=\mu-\alpha_1+\alpha_2+\varepsilon_{12}$	$-\alpha_{12}$
High	Low	1 1 0	0	$y_{21} = \mu + \beta_1 + \varepsilon_{21}$		1 1 -1	-1	$y_{21}=\mu+\alpha_1-\alpha_2+\varepsilon_{21}$	$-\alpha_{12}$
High	High	1 1 1	1	$y_{22} = \mu+\beta_1+\beta_2+\varepsilon_{22}$	$+\beta_{12}$	1 1 1	1	$y_{22}=\mu+\alpha_1+\alpha_2+\varepsilon_{22}$	$+\alpha_{12}$

the main effect for income at all levels of education. But for the interaction effect, either coding system will give an equivalent result.

Three Way Classification Without Interaction (THWOI)

Suppose, in addition to income and education, we are also interested in the effect on protest of occupational prestige, as measured by the Duncan Index.[7] Then the model can be written as

$$y_{ijk} = \mu X_0 + \beta_1 X_{1i} + \beta_2 X_{2j} + \beta_3 X_{3k} + \varepsilon_{ijk} \quad \text{for code B} \qquad (11)$$

$$y_{ijk} = \mu X_0 + \alpha_1 X_{1i} + \alpha_2 X_{2j} + \alpha_3 X_{3k} + \varepsilon_{ijk} \quad \text{for code A} \qquad (12)$$

where $i, j, k = 1, 2$; $X_0 = 1$; μ, α_i, and β_i are the regression coefficients for main effects; ε_{ijk} is a random error. The design matrix and cell models are shown in Table 2(i).

From Table 2(i) the estimators of regression coefficients can be computed as follows:

$$\begin{aligned} b_1 &= \tfrac{1}{4}(y_{2..}-y_{1..}), \quad b_2 = \tfrac{1}{4}(y_{.2.}-y_{.1.}) \\ b_3 &= \tfrac{1}{4}(y_{..2}-y_{..1}) \\ a_1 &= 1/8(y_{2..}-y_{1..}), \quad a_2 = 1/8(y_{.2.}-y_{.1.}) \\ a_3 &= 1/8(y_{..2}-y_{..1}) \end{aligned} \qquad (13)$$

The relationships between b's and a's in Equation 13 are $b_i = 2a_i$, $i = 1, 2, 3$.

The diagonal elements of $(X^TX)^{-1}$ with respect to b_i and a_i equal ½ and 1/8, respectively, therefore the Chi-square statistics are

$$\chi^2_{\beta_i} = b_i^2/\tfrac{1}{2} \quad \text{and} \quad \chi^2_{\alpha_i} = a_i^2/(1/8) \qquad (14)$$

Substituting a_i into $\chi^2_{\beta_i}$ gives

$$\chi^2_{\beta_i} = b_i^2/\tfrac{1}{2} = 4a_i^2/\tfrac{1}{2} = a_i^2/(1/8) = \chi^2_{\alpha_i}, \quad i = 1, 2, 3 \qquad (15)$$

It is obvious that the model without interaction will give equivalent statistics for both coding systems, except that the magnitude of b's are twice as large as a's. This result is consistent with the case of TWI.

Three Way Classification With 2-Factor Interaction (THWTI)

A model with three independent variables which are income, education, and the Duncan Index and their 2-factor interactions can be written as

$$y_{ijk} = \mu X_0 + \beta_1 X_{1i} + \beta_2 X_{2j} + \beta_3 X_{3k} + \beta_{12} X_{1i} X_{2j} + \beta_{13} X_{1i} X_{3k} + \beta_{23} X_{2j} X_{3k} + \varepsilon_{ijk} \quad \text{for code B} \qquad (16)$$

$$y_{ijk} = \mu X_0 + \alpha_1 X_{1i} + \alpha_2 X_{2j} + \alpha_3 X_{3k} + \alpha_{12} X_{1i} X_{2j} + \alpha_{13} X_{1i} X_{3k} + \alpha_{23} X_{2j} X_{3k} + \varepsilon_{ijk} \quad \text{for code A} \qquad (17)$$

where $X_0 = 1$; $i, j, k = 1, 2$; μ, α_i, β_i and ε_{ijk} are as defined in THWOI; α_{mn} and β_{mn} are 2-factor interaction regression coefficients, $m = 1, 2$; $n = 2, 3$ and $m < n$. The design matrix X and cell models are shown in Table 2(ii).

From Table 2(ii), the estimates of regression coefficients can be obtained as

$$\begin{aligned} b_1 &= y_{211} - y_{111}, \quad b_2 = y_{121} - y_{111} \\ b_3 &= y_{112} - y_{111} \\ b_{12} &= \tfrac{1}{2}[(y_{11.}+y_{22.}) - (y_{12.}+y_{21.})] \\ b_{13} &= \tfrac{1}{2}[(y_{1.1}+y_{2.2}) - (y_{1.2}+y_{2.1})] \\ b_{23} &= \tfrac{1}{2}[(y_{.11}+y_{.22}) - (y_{.12}+y_{.21})] \\ a_1 &= 1/8(y_{2..} - y_{1..}), \quad a_2 = 1/8(y_{.2.} - y_{.1.}) \\ a_3 &= 1/8(y_{..2} - y_{..1}) \\ a_{12} &= \tfrac{1}{4}[(y_{11.}+y_{22.}) - (y_{12.}+y_{21.})] \\ a_{13} &= \tfrac{1}{4}[(y_{1.1}+y_{2.2}) - (y_{1.2}+y_{2.1})] \\ a_{23} &= \tfrac{1}{4}[(y_{.11}+y_{.22}) - (y_{.12}+y_{.21})] \end{aligned} \qquad (18)$$

From Equation 18, the relationships of b's and a's become

$$\begin{aligned} a_1 &= \tfrac{1}{2}b_1 + \tfrac{1}{4}(b_{12} + b_{13}), \quad a_2 = \tfrac{1}{2}b_2 + \tfrac{1}{4}(b_{12}+b_{23}) \\ a_3 &= \tfrac{1}{2}b_3 + \tfrac{1}{4}(b_{12} + b_{23}) \\ a_{12} &= \tfrac{1}{4}b_{12}, \quad a_{13} = \tfrac{1}{4}b_{13}, \quad a_{23} = \tfrac{1}{4}b_{23} \end{aligned} \qquad (19)$$

The diagonal elements of $(X^TX)^{-1}$ with respect to b_i, b_{ij}, a_i and a_{ij} equal 1.5, 2.0, 1.25 and 1.25, respectively. Thus the Chi-square statistics are

TABLE 2 (i), (ii), (iii)

THREE WAY CLASSIFICATION DESIGN MATRIX AND CELL MODEL

Coding			(0, 1) B										
MODEL / VARIABLE			(iii) THWTTI; (ii) THWTI; (i) THWOI								(i) THWOI	(ii) THWTI	(iii) THWTTI
Income	Education	Duncan	X_0	X_1	X_2	X_3	X_{12}	X_{13}	X_{23}	X_{123}	Cell Model		
LOW	Low	Low	1	0	0	0	0	0	0	0	$y_{111} = \mu + \epsilon_{111}$		
	Low	High	1	0	0	1	0	0	0	0	$y_{112} = \mu + \beta_3 + \epsilon_{112}$		
	High	Low	1	0	1	0	0	0	0	0	$y_{121} = \mu + \beta_2 + \epsilon_{121}$		
	High	High	1	0	1	1	0	0	1	0	$y_{122} = \mu + \beta_2 + \beta_3 + \epsilon_{122}$	$+ \beta_{23}$	
HIGH	Low	Low	1	1	0	0	0	0	0	0	$y_{211} = \mu + \beta_1 + \epsilon_{211}$		
	Low	High	1	1	0	1	0	1	0	0	$y_{212} = \mu + \beta_1 + \beta_3 + \epsilon_{212}$	$+ \beta_{13}$	
	High	Low	1	1	1	0	1	0	0	0	$y_{221} = \mu + \beta_1 + \beta_2 + \epsilon_{221}$	$+ \beta_{12}$	
	High	High	1	1	1	1	1	1	1	1	$y_{222} = \mu + \beta_1 + \beta_2 + \beta_3 + \epsilon_{222}$	$+\beta_{12}+\beta_{13}+\beta_{23}$	$+\beta_{123}$
Coding			(-1, 1) A										
LOW	Low	Low	1	-1	-1	-1	1	1	1	-1	$y_{111} = \mu - \alpha_1 - \alpha_2 - \alpha_3 + \epsilon_{111}$	$+\alpha_{12}+\alpha_{13}+\alpha_{23}$	$+\alpha_{123}$
	Low	High	1	-1	-1	1	1	-1	-1	1	$y_{112} = \mu - \alpha_1 - \alpha_2 + \alpha_3 + \epsilon_{112}$	$+\alpha_{12}-\alpha_{13}-\alpha_{23}$	$+\alpha_{123}$
	High	Low	1	-1	1	-1	-1	1	-1	1	$y_{121} = \mu - \alpha_1 + \alpha_2 - \alpha_3 + \epsilon_{121}$	$-\alpha_{12}+\alpha_{13}-\alpha_{23}$	$+\alpha_{123}$
	High	High	1	-1	1	1	-1	-1	-1	-1	$y_{122} = \mu - \alpha_1 + \alpha_2 + \alpha_3 + \epsilon_{122}$	$-\alpha_{12}-\alpha_{13}+\alpha_{23}$	$-\alpha_{123}$
HIGH	Low	Low	1	1	-1	-1	-1	-1	1	1	$y_{211} = \mu + \alpha_1 - \alpha_2 - \alpha_3 + \epsilon_{211}$	$-\alpha_{12}-\alpha_{13}+\alpha_{23}$	$+\alpha_{123}$
	Low	High	1	1	-1	1	-1	1	-1	-1	$y_{212} = \mu + \alpha_1 - \alpha_2 + \alpha_3 + \epsilon_{212}$	$-\alpha_{12}+\alpha_{13}-\alpha_{23}$	$-\alpha_{123}$
	High	Low	1	1	1	-1	1	-1	-1	-1	$y_{221} = \mu + \alpha_1 + \alpha_2 - \alpha_3 + \epsilon_{221}$	$+\alpha_{12}-\alpha_{13}-\alpha_{23}$	$-\alpha_{123}$
	High	High	1	1	1	1	1	1	1	1	$y_{222} = \mu + \alpha_1 + \alpha_2 + \alpha_3 + \epsilon_{222}$	$+\alpha_{12}+\alpha_{13}+\alpha_{23}$	$+\alpha_{123}$

$$\chi^2_{\beta_i} = b_i^2/1.5,\ \chi^2_{\alpha_i} = a_i^2/1.25,\ \chi^2_{\beta_{ij}} = b_{ij}^2/2 \quad (20)$$

$$\chi^2_{\alpha_{ij}} = a_{ij}^2/.125 = (\tfrac{1}{4}b_{ij})^2/.125 = b_{ij}^2/2 = \chi^2_{\beta_{ij}}$$

where i, j = 1, 2.

These results indicate that the Chi-square statistics for the main effects a_i and b_i are not identical, but the 2-factor interaction terms are equivalent for both coding systems. Thus, if there is interaction in the three way classification model, the analysis should be interpreted carefully. For example, b_1 is only testing the income effect for the low education and low occupational prestige groups; b_2 is testing the education effect for the low income and low occupational prestige groups. To test main effects over all levels, the code A system should be used.

Three Way Classification With Two and Three Factor Interaction (THWTTI)

In addition to the main effects and the 2-factor interaction suppose 3-factor interaction are to be studied. Then the model can be written as

$$y_{ijk} = \mu X_0 + \beta_1 X_{1i} + \beta_2 X_{2j} + \beta_3 X_{3k} + \beta_{12} X_{1i} X_{2j} + \beta_{13} X_{1i} X_{2j} + \beta_{123} X_{1i} X_{2j} X_{3k} + \varepsilon_{ijk} \quad \text{for code B} \quad (21)$$

$$y_{ijk} = \mu X_0 + \alpha_1 X_{1i} + \alpha_2 X_{2j} + \alpha_3 X_{3k} + \alpha_{12} X_{1i} X_{2j} + \alpha_{13} X_{1i} X_{3k} + \alpha_{123} X_{1i} X_{2j} X_{3k} + \varepsilon_{ijk} \quad \text{for code A} \quad (22)$$

where i, j, k = 1, 2; μ, α_1, β_1, α_{mn} and β_{mn} are defined in TRWTI section; α_{123} and β_{123} are 3-factor interaction regression coefficients. The design matrix X and cell models are shown in Table 2(iii).

From Table 2(iii), the estimators of regression coefficients can be computed as

$$b_1 = y_{211} - y_{111},\quad b_2 = y_{121} - y_{111}$$
$$b_3 = y_{112} - y_{111},\quad b_{12} = (y_{221}+y_{111})-(y_{211}+y_{121})$$
$$b_{13} = (y_{212}+y_{111})-(y_{211}+y_{112})$$
$$b_{23} = (y_{122}+y_{111})-(y_{121}+y_{112})$$
$$b_{123} = (y_{222}+y_{121}+y_{211}+y_{112})-(y_{221}+y_{122}+y_{212}+y_{111})$$
$$a_1 = 1/8(y_{2..}-y_{1..}),\ a_2 = 1/8(y_{.2.}-y_{.1.})$$
$$a_3 = 1/8(y_{..2}-y_{..1}) \quad (23)$$
$$a_{12} = 1/8[(y_{22.}+y_{11.})-(y_{21.}+y_{12.})]$$
$$a_{13} = 1/8[(y_{2.2}+y_{1.1})-(y_{1.2}+y_{2.1})]$$
$$a_{23} = 1/8[(y_{.22}+y_{.11})-(y_{.12}+y_{.21})]$$
$$a_{123} = 1/8[(y_{222}+y_{121}+y_{211}+y_{112})-(y_{221}+y_{122}+y_{212}+y_{111})]$$

The relationship between a's and b's obtained from Equation 23 are as follows

$$a_1 = \tfrac{1}{2}b_1 + \tfrac{1}{4}(b_{12} + b_{13}) + 1/8(b_{123})$$
$$a_2 = \tfrac{1}{2}b_2 + \tfrac{1}{4}(b_{12} + b_{23}) + 1/8(b_{123}) \quad (24)$$
$$a_3 = \tfrac{1}{2}b_3 + \tfrac{1}{4}(b_{13} + b_{23}) + 1/8(b_{123})$$
$$a_{12} = \tfrac{1}{4}b_{12} + 1/8(b_{123}),\quad a_{13} = \tfrac{1}{4}b_{13} + 1/8(b_{123})$$
$$a_{23} = \tfrac{1}{4}b_{23} + 1/8(b_{123}),\quad a_{123} = 1/8(b_{123})$$

The diagonal elements of $(X^TX)^{-1}$ with respect to b_1, b_2, b_3, b_{12}, b_{13}, b_{23} and b_{123} are equal to 2, 2, 2, 4, 4, 4, and 8, respectively in code B. And those with respect all regression coefficients in code A are equal to 1/8. Thus, the Chi-square statistics become

$$\chi^2_{\beta_i} = b_i^2/2 \ \text{ is not equal to } \ \chi^2_{\alpha_i} = a_i^2/(1/8) \quad i = 1,2,3$$
$$\chi^2_{\beta_{ij}} = b_{ij}^2/4 \ \text{ is not equal to } \ \chi^2_{\alpha_{ij}} = a_{ij}^2/(1/8) \quad i=1,2;\ j=2,3;\ i < j \quad (25)$$

However,

$$\chi^2_{\beta_{123}} = b_{123}^2/8 = (8a_{123})^2/8 = a_{123}^2/(1/8) = \chi^2_{\alpha_{123}} \quad (26)$$

Equation 23 shows that b_i is only testing main effects in lower levels of the other two independent variables. For example, b_1 is testing the income effect in the group with low education and low occupational status. The 2-factor interaction b_{ij}, is testing interaction in the low level of the third factor. For instance, b_{12} is testing income and education effects in low occupational groups. The 3-factor interaction b_{123} is testing three factor interactions over all levels. In the code A system a_i, a_{ij} and a_{123} are presenting ordinary main effects, 2-factor and 3-factor interactions, respectively. In this model a_{123} and b_{123} give the same statistical result in the testing, but b_{123} is eight times larger than a_{123}.

Example of Application

Data used in this example are from a survey of parents in seven desegregated school districts throughout Florida.

On the basis of responses to questionnaire items by parents whose children attend desegregated public schools, two groups are classified:

(1) Those who did not protest against desegregation: Y_1

(2) Those who protested: Y_2.

To study the impact on protest of income, education and percent black in assigned schools, let the proportion, P, of protestors to non-protestors be the dependent variable, that is, $P=Y_2/(Y_1+Y_2)$. Education is X_1, income is X_2 and percent black is X_3. The least squares approach has been used to estimate the regression coefficients,

$$b = (X^TX)^{-1}X^TP \quad \text{and} \quad \chi^2_{\beta_i} = b_i^2/c_i$$

where c_i equals i^{th} diagonal element of $(X^TX)^{-1}$. The regression coefficients and Chi-square statistics with both coding systems are shown in Table 3. The proportion of protesters to non-protestors y_{ij}, which correspond to X_1 and X_2 are shown in Table 4, and y_{ijk} which corresponds to X_1, X_2 and X_3 are shown in Table 5. As can be seen from Table 3, the relationship between b's and a's for different models are demonstrated in numerical values which are consistent with the conclusions indicated in the techniques analysis section. Substituting y_{ij} of Table 4 into Equations 3 and 8, y_{ijk} of Table 5 into Equations 13, 18 and 23 and comparing results with b's and a's in Table 3, yields the following Equations 27, 28, 29, 30 and 31. The examples used in following equations are only demonstrated with a few cases. The entire cases of each model can be applied by using the similar techniques.

(i) (TWOI) Model

$$\tfrac{1}{2}(y_{2\cdot}-y_{1\cdot}) = \tfrac{1}{2}(.641-.590) = .026 = b_1$$

$$a_1 = \tfrac{1}{2}b_1 \qquad (27)$$

(ii) (TWI) Model

$$y_{21\cdot} - y_{11\cdot} = .280 - .239 = .041 = b_1$$

$$(y_{11}+y_{22})-(y_{12}+y_{21}) = (.239+.361)-(.351+.280) = -.031 = b_{12}$$

$$\tfrac{1}{4}(y_{2\cdot}-y_{1\cdot}) = \tfrac{1}{4}(.641-.590) = .013 = a_1$$

$$\tfrac{1}{4}b_{12} = a_{12} \qquad (28)$$

TABLE 3

Models	Variables	(0,1) B b	χ^2	(-1,1) A a	χ^2
TWOI	Education	.026	.0054	.013	.0054
	Income	.096	.0742	.048	.0742
TWI	Education	.041	.0069	.013	.0054
	Income	.112	.0500	.048	.0442
	Ed x Inc	-.031	.0019	-.008	.0019
THWOI	Education	.026	.0054	.013	.0054
	Income	.096	.0742	.048	.0742
	Black	.016	.0021	.008	.0021
THWTI	Education	.012	.0004	.013	.0054
	Income	.072	.0139	.048	.0742
	Black	-.053	.0075	.008	.0021
	Ed x Inc	-.031	.0019	-.008	.0019
	Ed x Blk	.059	.0070	.015	.0070
	Inc x Blk	.080	.0127	.020	.0127
THWTTI	Education	.024	.0012	.013	.0054
	Income	.084	.0141	.048	.0742
	Black	-.041	.0034	.008	.0021
	Ed x Inc	-.055	.0030	-.008	.0019
	Ed x Blk	.035	.0012	.020	.0127
	Ed x Inc x Blk	.048	.0012	.006	.0012

(iii) (THWOI) Model

$$\tfrac{1}{4}(y_{2\cdot\cdot}-y_{1\cdot\cdot}) = \tfrac{1}{4}(1.282 - 1.178) = .026 = b_1$$

$$a_1 = 1/8(b_1) \qquad (29)$$

TABLE 4

Education X_1	Income X_2 Low	High	$y_{i\cdot}$
Low	.239 (8)	.351 (8)	.590 (16)
High	.280 (8)	.361 (8)	.641 (16)
$y_{\cdot j}$	.519	.712	1.231

2 x 2 Table

(iv) (THWTI) Model

$$\tfrac{1}{2}[(y_{11\cdot}+y_{22\cdot})-(y_{12\cdot}+y_{21\cdot})] = -.031 = b_{12}$$

$$1/8(y_{2\cdot\cdot}-y_{1\cdot\cdot}) = 1/8(1.282 - 1.178) = .013 = a_1 = \tfrac{1}{2}b_1 + \tfrac{1}{4}(b_{12}+b_{13})$$

$$a_{12} = \tfrac{1}{4}b_{12} \qquad (30)$$

TABLE 5

Education X_1	Income X_2	X_3 Black % Low	High	Total
Low	Low	y_{111} .259	y_{112} .218	$y_{11\cdot}$ .477
	High	y_{121} .343	y_{122} .358	$y_{12\cdot}$ .701
	Total	$y_{1\cdot 1}$ .602	$y_{1.2}$ .576	$y_{11\cdot}$ 1.178
High	Low	y_{211} .283	y_{212} .277	$y_{21\cdot}$ .560
	High	y_{221} .312	y_{222} .410	$y_{22\cdot}$ .722
	Total	$y_{2\cdot 1}$ .595	$y_{2\cdot 2}$ .687	$y_{2\cdot\cdot}$ 1.282
TOTAL		$y_{\cdot\cdot 1}$ 1.197	$y_{\cdot\cdot 2}$ 1.263	$y_{\cdot\cdot\cdot}$ 2.460

2 x 2 x 2 Table

$y_{\cdot 1\cdot} = 1.037$ $\quad y_{\cdot 11} = .542$ $\quad y_{\cdot 12} = .495$

$y_{\cdot 2\cdot} = 1.423$ $\quad y_{\cdot 21} = .655$ $\quad y_{\cdot 22} = .768$

(v) (THWTTI) Model

$$y_{211} - y_{111} = .283 - .259 = .024 = b_1$$

$$(y_{221}+y_{111})-(y_{211}+y_{121}) = (.312+.259) - (.283+.343) = -.055 = b_{12}$$

$$(y_{222}+y_{121}+y_{211}+y_{112}) - (y_{221}+y_{122}+y_{212}+y_{111}) = (.410+.343+.283+.218) - (.312+.358+.277+.259) = .048 = b_{123}$$

$$1/8(y_{2\cdot\cdot} - y_{1\cdot\cdot}) = 1/8(1.282 - 1.178) = .013 = a_1 = \tfrac{1}{2}b_1 + \tfrac{1}{4}(b_{12} + b_{13}) + 1/8(b_{123})$$

$$1/8[(y_{22\cdot} + y_{11\cdot}) - (y_{21\cdot} + y_{12\cdot})] = 1/8[(.722 + .477) - (.560 + .701)] = -.0077 = a_{12} = \tfrac{1}{4}b_{12} + 1/8(b_{123})$$

$$a_{123} = 1/8(b_{123})$$

Clearly, the results in Equations 27 to 31 are also consistent with the analysis of techniques.

An advantage of using the (0, 1) coding system is the straightforward interpretation by which regression coefficients can be explained as the percentage-wise effect on the dependent variable. For instance, in the first model of Table 3

$$\hat{Y} = \hat{\mu} + (.026)X_1 + .096X_2 \qquad (32)$$

can be explained as follows: the proportion of protestors increased 2.6 percent from lower education to higher education, and 9.6 percent from lower income to higher income. However, in the model with interaction involved, the (-1, 1) coding system is more appropriate. For example, the second pair of models in Table 3 are

$$\hat{Y} = \hat{\mu} + .042X_1 + .112X_2 - .031X_1X_2 \text{ in code B} \qquad (33)$$

and

$$\hat{Y} = \hat{\mu} + .013X_1 + .048X_2 - .008X_1X_2 \text{ in code A} \qquad (34)$$

Equation 33 should be interpreted as follows: the proportion of protesters is 4.25 percent higher due to the education increase in the low income group, 11.2 percent higher due to the income effect in the low education group and 3.1 percent lower due to income and education interaction. To seek appropriate main effects, Equation 34 should be used. Equation 34 shows the education effect is 1.3 percent, the income effect is 4.8 percent and the interaction is 0.8 percent which is one-fourth of 3.1 percent in code B.

Summary and Conclusions

Two systems of dummy variables used to analyze contingency tables are coded either "0" and "1" or "-1" and "1". Based on five models of two way classification without interaction, two way classification with interaction, three way classification without interaction, three way classification with 2-factor interaction, and three way classification with two and three factor interactions, a contrast interpretation of the variables with and without interaction in these two coding systems has been mathematically explicated.

A survey of parents in seven desegregated school districts throughout Florida is used to demonstrate the application of the models. The results indicate that the conclusions obtained from the demonstrations are consistent with the interpretations given in the analyses of techniques. A major advantage of the (0, 1) coding system is the straightforward interpretation by which regression coefficients can be explained as the percentage-wise effect on the dependent variable.

Based on this study, the (0, 1) coding system is recommended to construct a design matrix for the model concerning only main effects, and the (-1, 1) coding system is suggested for situations concerning both main effects and interactions.

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